1985

1987

1989

1991

THE ESSEX ECHO

1995

MALCOLM OMAR HAWKSFORD EXAMINES ELECTRICAL SIGNAL PROPAGATION AND EXPLORES THE IMPLICATIONS FOR AUDIO CABLE PERFORMANCE

The matter of whether—and if so, how—speaker cables and interconnects can affect the sound of an audio system has vexed the audiophile community since Jean Hiraga, Robert Fulton, and others first made us aware of the subject in the mid-'70s. Most of the arguments since then have involved a great deal of heat but not much light. Back in August 1985, Professor Malcolm Omar Hawksford, Ph.D. (of the UK's University of Essex and a Fellow of the Audio Engineering Society) wrote an article for the British magazine Hi-Fi News & Record Review, *of which I was then Editor, in which he examined AC signal transmission from first principles. Among his conclusions was the indication that there is an optimal conductor diameter for audio-signal transmission, something that I imagined might lead to something of a conciliation between the two sides in the debate. Or at least when a skeptic proclaimed that "The Laws of Physics" don't allow for cables to affect audio performance, it could be gently pointed out to him or her that "The Laws of Physics" predict exactly the opposite.*

Well, I was wrong. Ten years later, as described in my recent "Wired!" essay (June '95, p.3), the "cables make a difference/no they don't" flame war continues unabated (though many of the more sonically successful "audiophile" cables tend to use conductors of the predicted optimal size). I asked Malcolm Omar, therefore, to revise and rewrite his 1985 article for Stereophile. *The essential math may look intimidating, but it's not as hard to grasp as it looks (you don't lose points for skipping it). The conclusions are both fascinating for those who use cables and essential reading for anyone who wishes to design audio cables.*

—John Atkinson

Audiophiles are excited. A special event has occurred that promises to undermine their very foundation and transcend "the event sociological"—a minority group now cite conductor and interconnect performance as a limiting factor within an audio system. The skeptical masses, however, remain content to congregate with their like-minded friends and make jokes in public about the vision of the converted. They are content to watch their distortion-factor meters confidently null at the termination of any old piece of wire (even rusty nails, it seems). Believing in Ohm's Law, they feel strong in their brotherhood—at least that's how it seemed back in 1985.

BUT THE REVOLUTION MOVES FORWARD...

This article examines propagation in cables—especially within conductive material—from the fundamental principles of electromagnetic theory. The aim is to consider mechanisms that form a more rational basis for an objective understanding of claimed sonic anomalies in interconnects, especially as the rumors about single-strand, thin wires persist.

Objective understanding relates to the choice of model used to visualize a phenomenon; thus we shall take a theoretic stance and commence with the work of Maxwell. The equations of Maxwell (see sidebar) concisely describe the foundation and principles of electromagnetism; they are central to a proper mathematical modeling of all electromagnetic systems. These equations are presented here in standard differential form and succinctly encapsulate the principles of electromagnetics, although further background can be sought from a wide range of texts. [1, 2, 3]

Maxwell's equations support a wave equation that governs the propagation of both the electric and magnetic fields in space and time, where the wave equation describing a propagating electric field $\overline{E}$ in a general lossy medium of conductivity σ (sigma), permittivity ε (epsilon), and permeability μ (mu) can be succinctly derived as follows:

Applying the vector operator curl on the Faraday equation,

$$\mathrm{curl}(\mathrm{curl}\,\overline{E}) = -\mathrm{curl}\left(\frac{\delta \overline{B}}{\delta t}\right) = -\mu \frac{\delta}{\delta t}\,\mathrm{curl}$$

Maxwell's Equations

Faraday's Law

$$\text{curl}\bar{E} = -\frac{\delta \bar{B}}{\delta t}$$

Ampere's Law

$$\text{curl}\bar{H} = \bar{J} + \frac{\delta \bar{D}}{\delta t}$$

Gauss's Theorem

$$\text{div}\bar{D} = \rho$$

No Magnetic Monopoles

$$\text{div}\,\bar{B} = 0$$

The constituent relationships that define electrical and magnetic material properties are:

$\bar{D} = \varepsilon_o \varepsilon_r \bar{E}$
$\bar{B} = \mu_o \mu_r \bar{H}$
$\bar{J} = \sigma \bar{E}$ (if σ is constant, then this equation represents Ohm's Law)
However, it is common to write $\varepsilon = \varepsilon_o \varepsilon_r$ and $\mu = \mu_o \mu_r$, thus $\bar{D} = \varepsilon \bar{E}$ and $\bar{B} = \mu \bar{H}$
where
$\bar{E}$, electric field strength, (volt/m)
$\bar{B}$, magnetic flux density, (tesla)
$\bar{D}$, electric flux density, (coulomb/m^2)
ρ, charge density, (coulomb/m^3)
σ, conductivity, $(\text{ohm-m})^{-1}$ (conductivity is the reciprocal of resistivity)
$\bar{H}$, magnetic intensity, (ampere-(turn)-m)
$\bar{J}$, current density, (ampere/m^2)
ε_o, permittivity of free space, (farad/m)
ε_r, relative permittivity
μ_o, permeability of free space, (henry/m)
μ_r, relative permeability
t, time, (second)
(The bar over some parameters indicates vector or directed quantities.)

Table 1 Copper Electrical Properties

$\sigma = 5.8 \times 10^7$ $(\text{ohm-meter})^{-1}$
$\varepsilon = 8.855 \times 10^{-12}$ farad/meter
$\mu = 4\pi \times 10^{-7}$ henry/meter

Table 2 Variation of Skin Depth & Velocity for copper with frequency

Frequency f, Hz	Skin depth δ, mm	Velocity v, m/s
50	9.35	2.93
100	6.61	4.15
1k	2.09	13.12
10k	0.66	41.50
20k	0.47	58.69

Substituting, $\bar{B} = \mu \bar{H}$ and for curl $\bar{H}$ from Ampere's law,

$$\text{curl}(\text{curl}\bar{E}) = -\mu \frac{\delta \bar{J}}{\delta t} - \mu \frac{\delta^2 \bar{D}}{\delta t^2}$$

Substituting $\bar{J} = \sigma \bar{E}$, $\bar{D} = \varepsilon \bar{E}$ and using the vector identity,

$$\text{curl}(\text{curl}\bar{E}) = \text{grad}(\text{div}\bar{E}) - \nabla^2 \bar{E}$$

The generalized wave equation in a conductive medium then follows as

$$\nabla^2 \bar{E} = \mu\sigma \frac{\delta \bar{E}}{\delta t} + \mu\varepsilon \frac{\delta^2 \bar{E}}{\delta t^2}$$

In this equation, ∇^2 is the vector Laplace operator, and we have assumed from Gauss's theorem that div $\bar{E} = \rho/\varepsilon = 0$ for a charge-free region. A similar equation can also be derived in terms of the $\bar{H}$ field, where, because of the symmetry of Maxwell's equations,

$$\nabla^2 \bar{H} = \mu\sigma \frac{\delta \bar{H}}{\delta t} + \mu\varepsilon \frac{\delta^2 \bar{H}}{\delta t^2}$$

In practice we shall consider only the $\bar{E}$ field, as the $\bar{H}$ field can be derived from Faraday's Law by integrating over time the vector curl $\bar{E}$, which reveals that at every point in space $\bar{E}$ and $\bar{H}$ are mutually at right angles, and also lie in a plane at right angles to the direction of propagation (fig.1).

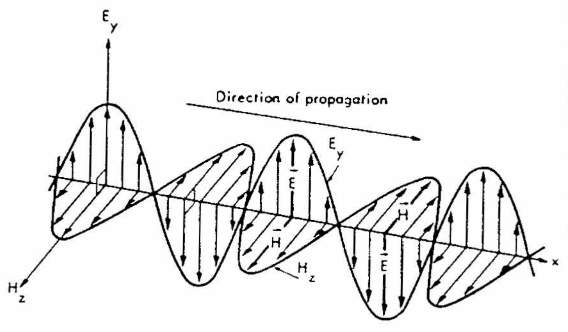

Fig.1 A propagating electromagnetic wave. The sinusoidally varying magnetic field ($\bar{E}$) is at right angles to the sinusoidally varying magnetic field ($\bar{H}$), and both are at right angles to the direction of propagation (from Dudley H. Towne's *Wave Phenomena*, Dover Publications, ISBN 0-486-65818-X, 1988 reprint of original 1967 book).

Consider a steady-state, sinusoidal electric field $\bar{E}$ propagating within a medium of finite conductivity where, because of the conversion of electrical energy into heat within a conductor, the traveling wave must experience attenuation. This suggests that a steady-state wave of sinusoidal form should decay exponentially as a function of distance z,

$$E = E_0 e^{-\alpha z} \sin(\omega t - \beta z)$$

α is defined as the attenuation constant, while the phase of the wave as a function of distance is determined by the phase constant $\beta = 2\pi/\lambda$, where λ (in meters) is the wavelength of the propagating field and $\omega = 2\pi f$, the angular frequency in radians/second.

An exponential decay is a logical choice, as for each unit distance the wave propagates it is attenuated by the same fractional amount. The electric field E is aligned to prop

REAL-WORLD CONDUCTORS HAVE FINITE CONDUCTIVITY, WHICH CAUSES THE GUIDED ELECTROMAGNETIC ENERGY TRAVELING IN THE DIELECTRIC TO SPILL OUT AND PROPAGATE INTO THE INTERIOR OF EACH CONDUCTOR.

gate in a direction z, where the direction of E is at 90° (right-angles) to z as shown in fig.2, where several phases are illustrated. Consequently, at a fixed point of observation z, E varies sinusoidally, while for constant time t, E plotted against z is a sinewave with exponential decay.

To check the validity of this solution, the function for E must satisfy the wave equation. This validation also enables the constants α and β to be expressed as functions of σ, ε, μ, and ω. However, because this substitution, although straightforward, is somewhat tedious, I will show only the initial working and then state the conclusion:

Substitute the assumed solution into the wave equation where, if propagation is assumed to take the direction z,

$$\frac{\delta^2 E}{\delta z^2} = \mu\sigma\frac{\delta E}{\delta t} + \mu\varepsilon\frac{\delta^2 E}{\delta t^2}$$

It follows that the function for E is a solution to the wave equation, provided that

$$\beta^2 - \alpha^2 = \mu\varepsilon\omega^2$$

$$\alpha\beta = \omega\mu\sigma / 2$$

Hence solving for α and β,

$$\alpha^2 = \frac{\mu\varepsilon\omega^2}{2}\left[(1 + (\frac{\sigma}{\varepsilon\omega})^2)^{0.5} - 1\right]$$

$$\beta = \frac{\omega\mu\sigma}{2\alpha}$$

where the constants α and β that govern the velocity and attenuation of the propagating field can be expressed in terms of the angular frequency ω and the parameters μ, ε, and σ, which are documented for most materials. (α and β are sometimes expressed as a complex number in terms of the propagation constant γ, where $\gamma = \alpha + j\beta$.)

Okay, so many of you may not have followed the details of the mathematics. Don't worry–it's really only important to appreciate the high-level procedures, namely:

• Commence with Maxwell's equations, from which is derived the generalized wave equation for propagation in a lossy material.
• Guess at a logical solution for a sinusoidal plane wave, knowing that Fourier analysis allows a generalization to more complicated waveforms (at least for a linear medium).
• Show that the chosen solution satisfies the wave equation, where the propagation constants α and β follow as functions of the material constants μ, ε, σ, and the angular frequency ω.
• The velocity of propagation v (in meters/s) is expressed in terms of ω and β as

$$v = f\lambda = 2\pi f(\frac{\lambda}{2\pi}) = \frac{\omega}{\beta}$$

We can now classify materials into good conductors (*eg*, metals) and poor conductors (lossy dielectrics, or "insulators"), although this demarcation is frequency-dependent.

Poor Conductors: These are dielectric materials with very low conductivity. As σ is so small, σ is inconsequential compared with εω. Consequently, α approaches 0 and the wave experiences minimal attenuation. This condition applies to propagation in both free space and low-loss dielectrics, where the velocity of propagation can be shown to be

$$v = \frac{1}{\sqrt{(\mu\varepsilon)}} = \frac{300.10^8}{\sqrt{(\mu_r\varepsilon_r)}} \text{ m/s}$$

For "free space," the relative permeability and permittivity, μ_r and ε_r, are both equal to 1; v is equal to c, the velocity of light. This is the "fast wave" and justifies the common comment that, for audio interconnects, the velocity of propagation within the dielectric is so high that signals respond virtually instantaneously across the length of the cable. However, we must be more cautious when discussing EMC-related problems and digital interconnects communicating high-speed data where this velocity becomes a significant factor.

Good Conductors (such as copper): Assume ω is very much smaller than σ/ε, which for copper implies the frequency $f < 1.04.10^{18}$Hz (see Table 1). At audio frequencies, therefore, copper is an excellent conductor, where α and β approximate to $\alpha = \beta = \sqrt{(\mu\omega\sigma/2)}$–*the values for α and β are identical for a good conductor.*

The velocity of propagation in a lossy material follows from $v = \omega/\beta$, whereby $v = \sqrt{(2\omega/\mu\sigma)}$. *This is very much lower than that for a material with low conductivity.*

For copper (using the material parameters in Table 2), α, β, and v are given by

$$\alpha = \beta = 15.13\sqrt{f} \text{ and } v = 0.415\sqrt{f}$$

Note the frequency dependence of α, β, and v–*All are significant at audio frequencies!!!* At 1kHz, the velocity is just 1/25 of the velocity of sound in air!

Skin depth

Skin depth δ is defined as the distance (in meters) an electromagnetic wave propagates for its value to be attenuated by a factor of 1/e, where e is the number used as the base for natural logarithms.[1] As e = 2.71828…, 1/e = 0.3679…, or –8.69dB. It can be seen from the traveling wave solution that $E = E_0 e^{-\alpha z}\sin(\omega t - \beta z)$, then for $z = \delta$, $e^{-\alpha\delta} = e^{-1}$, whereby

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\mu\omega\sigma}}$$

That is, the skin depth δ is the reciprocal of the attenuation constant α. However, skin depth is only a definition. You

1 To those fascinated by numbers I heartily recommend two essential books: Eli Maor's *e–The Story of a Number*, Princeton University Press, ISBN 0-691-03390-0, 1990; and Petr Beckmann's *A History of π (Pi)*, St. Martin's Press, ISBN 0-312-38185-9, 1974 (Third Edition).–JA

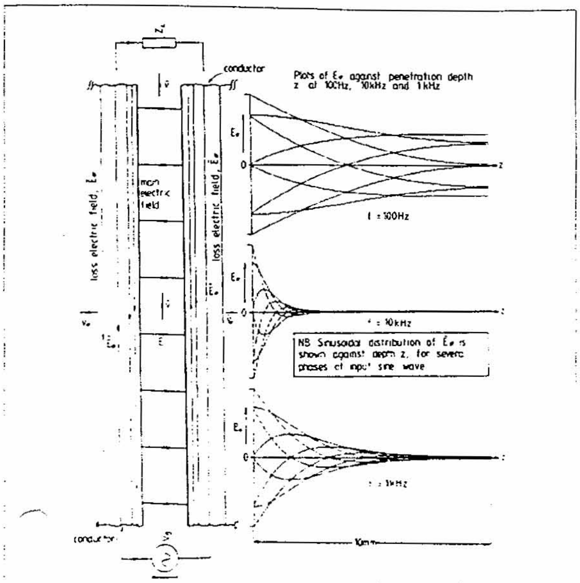

Fig.2 Approximate relationship of propagating field $\overline{E}$ to loss field $\overline{E}\sigma$. $\overline{E}\sigma$ is sinusoidal and is shown at 100Hz, 1kHz, and 10kHz for a conductor of width 10mm.

should appreciate that the fields still exist when the propagation distance z is greater than the skin depth δ, even though they are attenuated. For example, for an attenuation of just over 30dB the propagation distance is 3.5δ. Also, at $z = \delta$, the phase (δz) of E has changed by 1 radian, or 57.3°—a far-from-negligible figure. Table 2 gives examples of calculations of skin depth and velocity against frequency for copper, where $\delta = (15.13\sqrt{f})^{-1}$.

The low value of velocity is directly attributable to the high value of conductivity for copper, $\sigma = 5.8 \times 10^7$ (ohm-m)$^{-1}$. For comparison, for silver, $\sigma = 6.14 \times 10^7$ (ohm-m)$^{-1}$. For aluminum, $\sigma = 3.54 \times 10^7$ (ohm-m)$^{-1}$.

These results suggest a copper wire with a maximum diameter of between 0.5mm and 1mm is optimum if a uniform current flow across the conductor is to be maintained over the audioband. However, there are additional factors to consider: An electric field traveling within copper has a low velocity and experiences high attenuation that results in skin depths that are significant in audio interconnect design. The frequency dependence of δ (as well as α and β) should not be underestimated: the copper acts as a spatial filter where the field patterns within the conductor for a broad-band signal exhibit a complicated form (again, see fig.2). Now introduce either or both a spatially distributed non-linearity or a discontinuous conductivity as previously discussed in *Hi-Fi News & Record Review*,[2] and the hypothesis that cables can exhibit performance defects becomes more plausible.

Let us continue with the model development. Electromagnetic theory shows that the two conductors of a cable act as "guiding rails" for the electromagnetic energy that propagates principally through the space between the conductors, where the electrical currents in the wires are directly a result of the field boundary conditions at the dielectric/wire interface, following from Gauss's Theorem.

This may prove a more difficult conceptual step for those accustomed to lumped circuits and simple current-flow models. However, a guided-wave model is supported by electromagnetic theory irrespective of cable geometry. Only the field patterns vary, depending upon the conductor shape and their spatial relationship. This theoretical model is not new—it has been widely accepted and practiced by engineers for many years.

As shown in fig.1, a propagating electromagnetic wave consists of an oscillation of energy between the magnetic and electric fields, these akin to kinetic and potential energy in a mechanical system. It is helpful to think of free-space dielectrics and conductors as distributed inductance-capacitance-resistance networks. For example, in a coaxial cable (fig.3), the electric field is everywhere radial, while the magnetic field forms concentric circles around the inner conductor (Ampere's Circuital Law). It is important to note, as commented upon earlier, that the $\overline{E}$ and $\overline{H}$ fields are both spatially at right angles to each other and to the direction of propagation, which is along the axis of the cable. This is a direct result of Maxwell's Equations.

Power flow

In an electromagnetic system, the "power flow" is a directed or vector quantity and is represented as a power-density function $\overline{P}$ (watt/m^2) called the Poynting Vector, where $\overline{P} = \overline{E} \times \overline{H}$ (with x being the vector crossproduct rather than the conventional multiplication symbol). For a coaxial cable, $\overline{P}$ is directed axially, indicating that energy is propagated *along* the cable. Integrate $\overline{P}$ over a cross-section of area (between the conductors) and the result is the total power carried by the cable.

The expression for $\overline{P}$ can be compared with power calculations in lumped electrical systems, where P (power) = V (voltage) x I (current) (*ie*, V is equivalent to the $\overline{E}$ field, I to the $\overline{H}$ field).

If we assume that the two conductors of the coaxial cable are ideal (where σ approaches infinity), then all the electromagnetic energy flows through the dielectric (insulator)

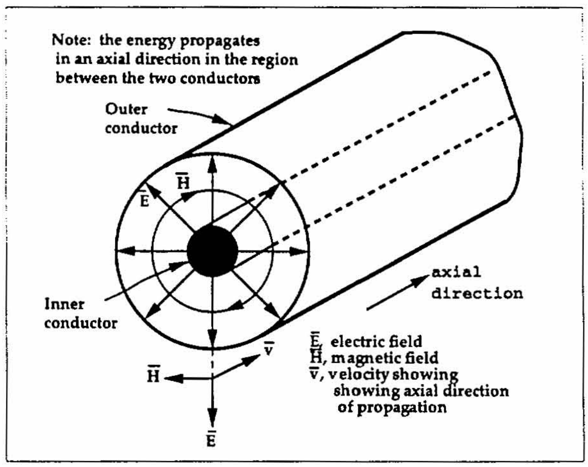

Fig.3 Cross-section of coaxial cable showing radial $\overline{E}$ field and circumferential $\overline{H}$ field.

because the electric field within the conductors must be zero. The $\overline{E}$ field does not penetrate the conductors, the skin depth is zero (check back to the expression for δ), and the conductors act as perfect reflectors (that's why a mirror is coated with a good conductor). In this ideal case, there is only a surface current on each conductor to match the

2 Martin Colloms, "Crystal Linear and Large," *HFN/RR*, November 1984, Vol.29 No.11, pp.47–49.

boundary condition for the tangential magnetic field $\bar{H}$ at the dielectric/conductor interface (see Skilling [2]). You should visualize a wave traveling in an axial direction within the dielectric and being guided by the conductors, where the electric and magnetic fields are both at right angles to each other and to the direction of propagation along the axis of the cable.

However, this perfect example is unrealistic. All practical conducting materials have a finite conductivity σ. They must inevitably exhibit an energy loss because, at the molecular level, collisions convert electrical energy into heat.

As the wave propagates through the dielectric, a study of the boundary condition reveals that the radial electric field $\bar{E}$ is not quite at 90° to the conductor surface. This is a direct consequence of the finite conductivity producing a longitudinal component, $\bar{E}_\sigma$ which is the voltage drop (in V/m) along the wire. The wave no longer takes the shortest path through the dielectric surrounding the conductor, but appears to travel more slowly at the conductor surface (rather in the way flowing water close to a river bank gives a c[illegible]d velocity profile).

At each dielectric-to-conductor interface, a refracted field results within the conductor (the longitudinal $\bar{E}_\sigma$ field) which proceeds to propagate radially at virtual right angles to the axis of the cable into the interior of the conductor. This field we shall call the "loss field." Consequently, the majority of the electromagnetic energy propagates in a near-axial direction within the dielectric, but a much-reduced loss field propagates almost radially into each conductor.

The electric field $\bar{E}_\sigma$ is oriented axially along the length of the conductor while the H field remains circumferential, curling around the internal longitudinal current. The Poynting Vector of the loss field is therefore directed radially into the conductor. It is the propagation of this loss field in a radial direction that is controlled by the material parameters of the conductors (μ, σ, ε) and is ultimately attenuated by conversion to heat. It is here that the story becomes more relevant to audio.

To summarize, real-world conductors have finite conductivity, which causes the guided electromagnetic energy traveling in the dielectric between the conductors to spill out and propagate into the interior of each conductor. Although the main component of energy propagates rapidly within the dielectric along the axis of the cable, the energy constituting the loss field that enters the conductor across the boundary propagates much more slowly (see Table 2), being determined by the frequency-dependent parameters α and β. In a perfect conductor, a current traveling only in the surface would match the boundary conditions for the axial propagating field. (Maybe you recall from Gauss's Theorem that the lines of the displacement vector $\bar{D}$ must terminate on a surface-charge density.) For a lossy conductor, however, it is the loss wave within the conductor that determines the current within the copper. The depth to which the E field, hence the conduction current, penetrates the conductor is expressed as the skin depth δ. We would therefore expect a complicated current distribution throughout the volume of the conductor, as suggested in fig.4.

The "Ohm's Law" of Maxwell's Equations states $\bar{J}_\sigma = \sigma\bar{E}_\sigma$, which indicates that a conduction current density $\bar{J}_\sigma$ is induced axially within the conductors due to the internal electric field $\bar{E}_\sigma$, of the loss wave. This axial current is the

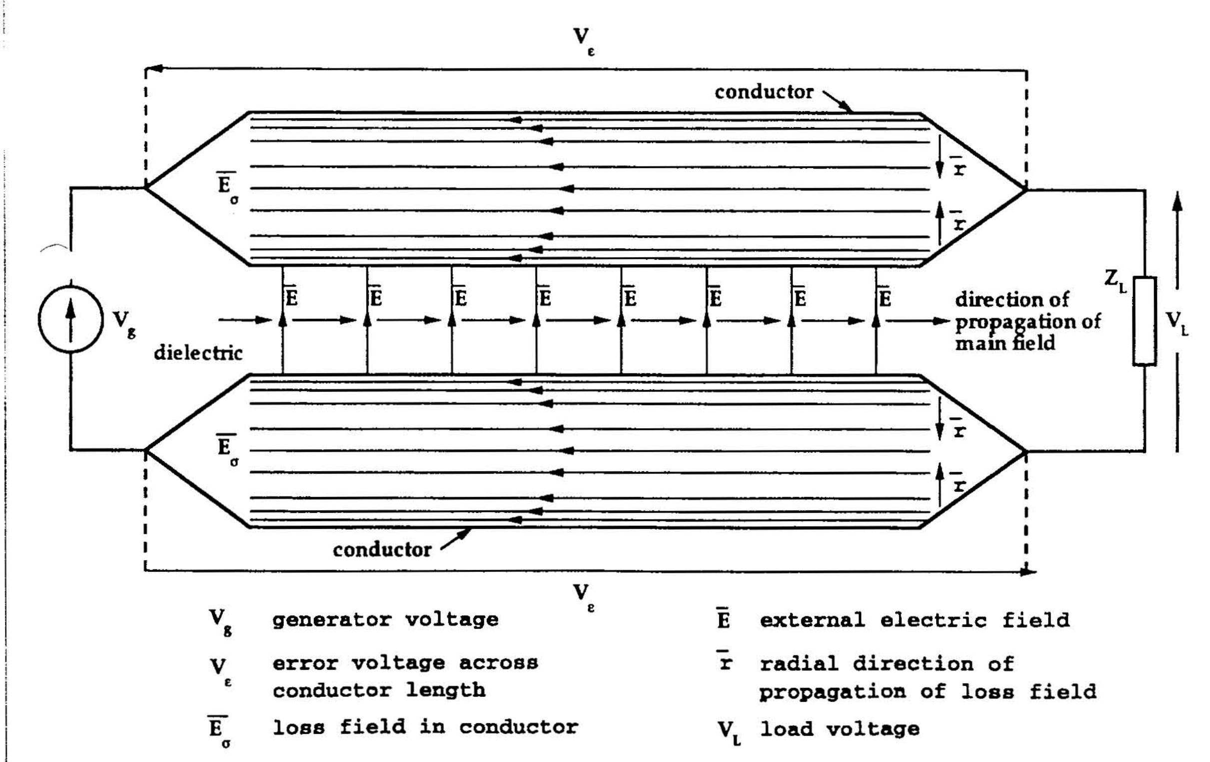

Fig.4 Basic field relationships and direction of propagation of main external field and internal loss field.

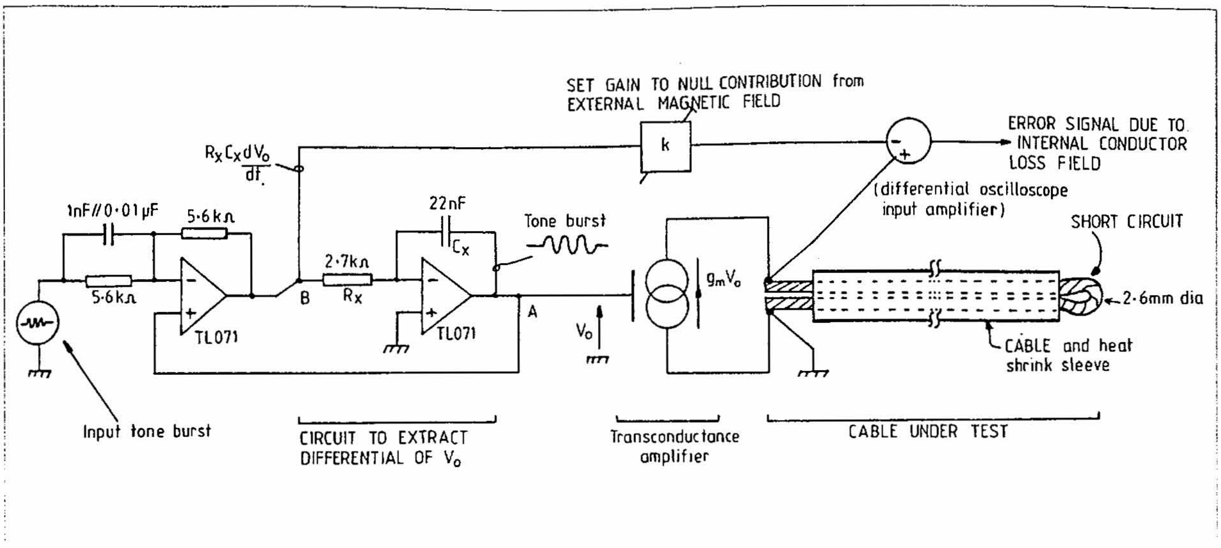

Fig.5 Measurement system for extracting error signal due to internal loss field within a conductor exhibiting significant "skin" depth.

current we normally associate with cables—the model we have developed is compatible with the more usual observations of cable behavior. Since the electromagnetic energy of the loss wave propagates principally in a radial direction and enters the conductor over its surface area, the current density (proportional to $\overline{E}_\sigma$) is greatest at the surface, and decays as the field propagates into the conductor interior. It is this reason why a conductor experiences a "skin effect" on the outer region rather than the converse situation of current concentrated near the center of the conductor.

Timing problems

One of the more informative parameters that we can calculate is the time T_δ for the sinusoidal loss field $\overline{E}_\sigma$ to traverse a distance δ within a good conductor, where, since

$v = \omega/\beta = \omega/\alpha = \omega\delta$

then

$T_\delta = \delta/v = 1/\omega$, as long as δ is much greater than ωε.

For example, consider a copper bar whose diameter is very much greater than its skin depth:

δ = 0.66mm at 10kHz: T_δ = 15.9µs
δ = 2.09mm at 1kHz: T_δ = 0.159ms
δ = 6.61mm at 100Hz: T_δ = 1.59ms

For conductor diameters that are very much greater than the skin depth, T_δ increases with decreasing frequency—there is energy storage; it is a memory mechanism. The model we have developed attempts to show that a good but unnecessarily large conductor influences transient behavior by time-smearing a small fraction of the applied signal by a significant amount.

Consider the cable construction shown in fig.4, where the generator inputs a sinewave for a time very much greater than T_δ, enabling the steady-state to be attained. The $\overline{E}$ field between the conductors responds rapidly to the applied signal, as the velocity in the dielectric between the conductors is very high. (We are assuming here a terminating load to the cable, so there is a net energy flow through the dielectric.) As the wavefront progresses, a radial loss wave propagates into each conductor, where the $\overline{E}_\sigma$ field is aligned in an axial direction.

Now allow the applied signal to be suddenly switched off. The field between the conductors collapses rapidly, thus cutting off the signal energy being fed radially into the conductors. However, the low velocity and high attenuation of the loss wave represents a lossy-energy reservoir, where the time for the wave to decay to insignificance as it propagates into the interior of the conductor is nontrivial by audio standards. The $\overline{E}_\sigma$ field within the conductor can be visualized as many "strands" of the $\overline{E}$ field, as shown in fig.4.

The error voltage, V_{int} appearing across the ends of each thread due to the internal conductor impedance, is calculated by multiplying the $\overline{E}_\sigma$ field by the cable length L (though strictly this should be an integral performed over each elemental length of the cable). Because the field propagates slowly, this summation is actually an average taken over a time window that extends over a short history of the loss field. Consequently, when the generator stops, the error signal across each conductor does not collapse instantaneously. Instead, the conductor momentarily becomes the generator, and a small time-smeared transient residual occurs as the locally stored energy within each conductor dissipates to insignificance. To this voltage must be added the generally more dominant induced voltage, V_{ext} arising from the changing external magnetic field. Together, they constitute an error voltage $V_\varepsilon = V_{int} + V_{ext}$. Assuming the two conductors are symmetrical, then the load voltage V_L is related to the generator voltage V_g by $V_L = V_g - 2V_\varepsilon$.

Although V_ε is very much smaller than V_g, the error voltage can take on a complicated and time-smeared form that in practice is both a function of the conductor geometry, cable characteristic impedance, generator source impedance, and load impedance, as all these factors govern the propagation of both the main field $\overline{E}$ and loss field $\overline{E}_\sigma$.

In practice, unless the cable is terminated in its characteristic impedance, the external field $\overline{E}$ residing in the dielectric will traverse the length of the cable rapidly back and forth many times before establishing a pseudo-steady state. Even an optimal load termination implies a significant loss field in the conductors. This argument would suggest that for non-power-carrying interconnects, it is better to terminate the generator end of the cable in the characteristic impedance, leaving the load as a high impedance. The $\overline{E}$ field is then rapidly established in the dielectric without either multiple reflection along the cable length, or a finite power

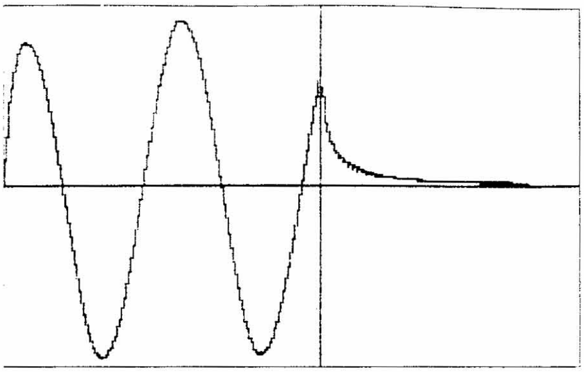

Fig.6 Predicted error due to internal loss field of conductor operating in skin depth-limited region.

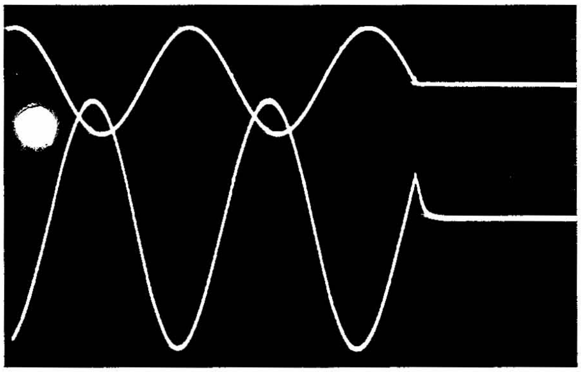

Fig.7 Measured error due to internal loss field of conductor operating in skin depth-limited region. Note that input and output waveforms are not on the same scale, and that time dispersion is proportional to the tone period in the skin depth-limited region, meaning that the time scale is immaterial.

flow to the load spilling out a loss wave into the conductors.

Theory vs. measurement

Let us now consider an experiment with measurements performed on an actual cable to see whether we can observe the error predicted from theory when the signal stops abruptly. Fig. 5 shows a measurement system used to determine the error voltage due to the internal fields of a folded loop of steel . (The diameter of the wire is 2.6mm, approximately 0.1".) The folded conductors are insulated but squeezed closely together to minimize the external magnetic field, hence the cable inductance. However, there will still be a residual external magnetic flux, so electronic compensation is made using a differentiator. This enables the signal to be subtracted from the voltage produced at the ends of the loop.

Note that the series inductance due to the external magnetic field produces a voltage proportional to the rate of change of magnetic flux (in turn proportional to the current). By subtracting the correct proportion of this voltage, therefore, the error voltage due to the internal fields in the conductor will be revealed. This supports the idea of measuring the actual error of a specific mechanism rather than letting it become swamped by a higher-level component.

Fig.6 shows a computer simulation of a toneburst that has been modified by a transfer function similar to that occurring in the cable. A similar measured result is shown in fig.7. In the steady state, a 45° phase shift occurs. However, after the tone is switched off, a dispersive component is revealed. This latter period is where the energy contained within the conductors decays and is constrained by the low velocity of propagation within a good conductor.

It must be noted, of course, that the error voltage induced by the changing external field dominates, so this result would be largely masked. (The greater the conductor spacing, the greater the masking.) With loudspeaker cables (greater than, say, 2mm in diameter) that are terminated close to their characteristic impedance, the external fields will contribute minimal error. However, the loss field still produces time dispersion. If the error voltage is to be correctly predicted, skin depth must be included, a simple resistive model being inaccurate. I am not trying to say that this effect is necessarily significant, only that an error component is predicted by our theory and is shown by the measurements to exist.

Multiple conductors

At this juncture, it is interesting—may I be so bold as to say *fun*?—to conjecture both what happens to an electromagnetic loss field propagating through a copper conductor when it encounters an abrupt discontinuity in conductivity, and whether this has any correlation with defects in copper that are attributable to crystal boundary interfaces.

Consider a long transmission line terminated in its characteristic impedance. Electromagnetic energy entering the line will propagate in a uniform manner, finally being absorbed in the load (just as with a VHF aerial cable which is terminated in 75 ohms). However, if the termination is in error, then a proportion of the incident energy is reflected back along the cable toward the source. In extreme cases, where there is either an open or a short-circuit load, then all the incident energy is reflected, though for a short circuit the sign of the $\bar{E}$ field is reversed upon reflection, thus canceling the electric field in the cable and informing the source that there is a short-circuit termination.

The point to observe is that a discontinuity in the characteristic impedance results in at least partial reflection at the discontinuity, which will distort the time-domain waveform. This reflective property of a change in characteristic impedance is used to locate faults in long lengths of cable by using time-domain reflectometry—a pulse is transmitted along the cable, and the return times of the partial echoes from each discontinuity indicate the location of the fault.

Similarly, for a wave traveling in copper, an impedance discontinuity (read here micro-discontinuity) leads to partial reflection centered on the discontinuity. This effect must be compounded with an already dispersive propagation—*ie*, different frequencies propagate at different velocities—thus time-smearing the error signal or loss wave in the conductor.

Now let's play to the gallery...This observation gives insight into the possible effects of crystal boundaries within copper, where each boundary can be viewed as a discontinuity in conductivity and corresponds to zones of localized partial reflection for the radial loss field.

Note, however, that this property is not necessarily nonlinear. We do not have to invoke a semiconductor-type nonlinearity to identify a problem; we're probably talking here of mainly linear errors. So we would not necessarily expect to observe either a significant reading on the distortion-factor meter, or modulation noise or sidebands in high-resolution spectral analysis, for steady-state excitation. However, just as with loudspeaker measurements (which often show non-minimum-phase behavior), amplitude-only response measurements do not give a complete representation of stored energy and time-delay mechanisms. We would

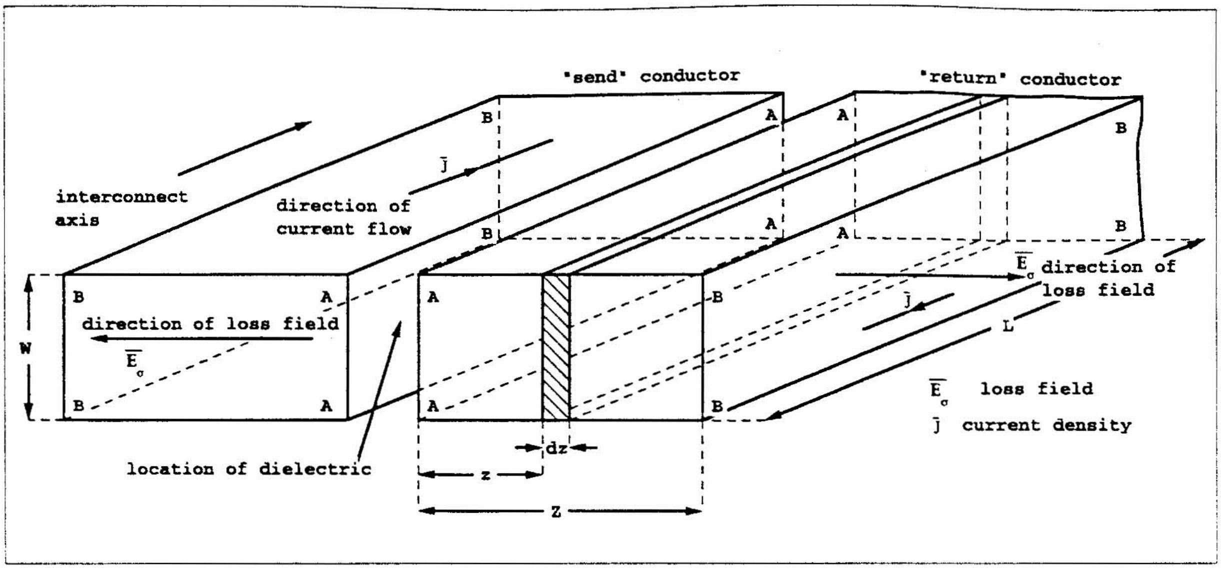

Fig.8 "Contrived" conductor geometry for skin-depth calculation.

require very careful measurements directly of the errors for both amplitude and phase, or of impulse response in the time domain.

Following the comments on the error function in an earlier "Essex Echo" article,[3] direct measurements of the output signal will in general yield insufficient accuracy to allow a true estimate of the system error. This point is worth thinking about: Ideally, we need to assess the actual current distribution in the conductors; or, more realistically, to measure the conductor error directly.

Real-life boundaries

The final stage in the development of our model is to account for copper conductors of finite thickness, where the thickness may well be much *less* than the skin depth. Just as a wave traveling in air is reflected when confronted by a short-circuit, so a wave that encounters an open circuit (*eg*, a copper/air boundary) while traveling in a conductor also undergoes reflection and therefore passes back into the conductor, undergoing further attenuation as it does so. However, the boundary conditions require a reversal of the magn[illegible] field. Provided the thickness of the conductor is much less than the skin depth, therefore, the incident and reflected $\overline{H}$ fields nearly cancel, and the conductor exhibits a lower internal magnetic field. Consequently there is predominantly an axial electric field and corresponding axial conduction current, the conductor behaves nearly as a pure resistor, and the magnetic field—hence the inductive component—is reduced to the pseudo-static case. The current distribution is nearly uniform. The conductor has lost its memory.

Impedances

Because we are dealing with a propagation phenomenon, the traveling-wave model can be used to evaluate the component of conductor impedance due to the internal fields. Since non-Litz conductor radii range at most to a few millimeters, the internal wave will undergo reflection at the conductor boundary due to the gross impedance mismatch between dielectric and conductor. I must own up that this area of the analysis is more difficult; at best, the results are approximate due to the cylindrical geometry. Consequently, when you write your letters of complaint, please address them directly to John Atkinson!

A contrived rectangular conductor geometry is shown in fig.8. Let the current density be J_0 at boundary A, where the loss wave propagates in the z direction; note that the current is aligned with the elemental conductor length ΔL. As the wave propagates from boundary A toward boundary B, it undergoes attenuation as the conductor has high conductivity. At boundary B there is total reflection due to the impedance mismatch. Similarly, the returned wave is re-reflected at A, and so on at subsequent boundary encounters. Considering these multiple-boundary reflections and noting the attenuation and phase over a length z is $e^{-\gamma z}$,

$$J(z) = J_0\left[e^{-\gamma z} + e^{-\gamma(2Z+z)} + K\right] = J_0\left[\frac{e^{-\gamma z} + e^{-\gamma(2Z-z)}}{1 - e^{-\gamma 2Z}}\right]$$

The total current I is calculated by integrating J(z) over the conductor cross-section,

$$I = W\int_{z=0}^{Z} J(z)dz = \frac{WJ_0}{\gamma}$$

For a good conductor, $\alpha = \beta = 1/\delta$ and $\gamma = \alpha + j\beta = (1+j)/\delta$, whereby

$$J_0 = I(1+j)/(W\delta)$$

The voltage ΔV appearing across an axial length of conductor ΔL is calculated from the surface electric field E; however, noting $J = \sigma E$, then $\Delta V = E\,\Delta L = J(z{=}0)\,\Delta L/\sigma$, and substituting from the hyperbolic family of functions,

$$Z_{int}(Z) + \frac{(1+j)}{W\sigma\delta}\coth(\gamma z)$$

Although contrived, the final result demonstrates a reasonable approximation of the internal impedance, assuming a nongranular conductivity. As a final step, transform the expression for $Z_{int}(Z)$, noting

3 Malcolm Omar Hawksford, "The Essex Echo: on errors, low feedback and fuzzy distortion," *HFN/RR*, September 1984, Vol.29 No.9, pp.37–41.

At a diameter of around 0.8mm, the conductor becomes closer to a low-valued ideal resistor at audio frequencies.

$$\gamma = (1+j)\sqrt{\left[\frac{\mu\omega\sigma}{2}\right]} = \frac{1+j}{\delta}$$

and $R_{int} = Z_{int}(Z)$ at zero frequency; *ie*, the DC resistance/meter, $R_{int}(Z) = (WZ\sigma)^{-1}$

$$Z_{int}(Z) = R_{int}\left[\frac{Z(1+j)}{\delta}\coth\left(\frac{Z(1+j)}{\delta}\right)\right]\text{ohm / meter}$$

This final expression allows an independent selection of conductor DC resistance and conductor radius (in a multistrand construction), assuming the skin depth δ is known (δ is a function of ω, μ, and σ).

Although there is a limit on the maximum conductor radius for a given DC resistance/meter and material conductivity, for a cylindrical conductor of radius Z which is equal to or less than $(\pi\sigma R_{int})^{-0.5}$, a Litz cable may be assembled from a number of paralleled and individually insulated conductors. With such a construction, the transition frequency at which skin depth becomes significant can be controlled. At low frequencies, where δ is greater than Z, the internal impedance of a conductor is substantially resistive. For δ smaller than Z—the wire is greater in diameter than the skin depth—the internal impedance is proportional to the square root of the frequency, with a phase shift approaching 45°.

The critical frequency, f_c, follows approximately from the expression for α ($\alpha = 1/\delta = 1/Z$) where $f_c = 1/(\pi\mu\sigma Z^2)$. For a cable with a diameter of 0.8mm, therefore, f_c = 27kHz. (This result is approximate, as the cylindrical geometry has not been fully accounted for in the analysis. [*Maybe it might turn out that a cylindrical conductor is less optimal than a rectangular, something that has been postulated by both Madrigal and TARA Labs.—Ed.*]) We therefore conclude that, for the internal impedance taken over the audio band, a thin conductor behaves as a resistor, whereas a thick conductor has a complex (frequency-dependent) impedance.

In this latter discussion we have interpreted our model in the steady-state and as a lumped impedance. However, we should not lose sight of the time-domain model and the extension to a discontinuous or granular conductivity. As observed in the earlier "Echo" article,[4] steady-state analysis, though correct, can limit our appreciation of a system. We would not expect to observe all anomalies easily on steady-state tests. As they can be hidden from view when the errors are of low level, such tests can be insensitive. Alternatively, direct measures of errors should be sought rather than attempting to extract them from a dominant signal.

In developing this model we have concentrated on the loss mechanism inherent in the conductors. We have not discussed the characteristic impedance observed at the input of the interconnect. This is a direct result of the amount of energy stored in the electric and magnetic fields needed to "fill" the cable—*ie*, the propagating energy within the cable system.

Ultimately, the energy loss in the interconnect is a function of the characteristic impedance, the cable length, and the load termination, as these directly influence the loss field, hence the conductor current, hence the voltage across the cable length. The load impedance that terminates the line is mapped into the interconnect error mechanisms. This latter observation is very important with loudspeaker loads, many of which are nonlinear. Much of the error attributable to loudspeaker cable is a consequence of the nonlinear load impedance acting with the series impedance of the cable—this must include the effect of skin depth.

The detailed characteristics of the dielectric material are also important, as the model shows that the dielectric supports the majority of the signal during its transportation along the cable (which can take many passes if the cable is not optimally terminated). Dielectric loss has been cited as a contributory factor, which can be modeled as an equivalent frequency-dependent but low-conductivity σ_d where $\sigma_d = \omega\varepsilon$ (power factor). Power factors vary from typically 0.0005 to 0.05 (see Skilling [2]). The attenuation and phase constants then follow, as $\sigma_d = 0.05\omega\varepsilon\sqrt{\mu}$ (Power factor) and $\beta_d = \varepsilon\sqrt{\mu}$. However, it is difficult to see how these results affect audio cables from this simplistic appraisal. A more detailed study of the permittivity of dielectrics, some of which are nonlinear, is required.

Conclusions

The basic elements of our model are now complete, wherein we propose that the internal loss fields that propagate within the conductors are at least partially responsible for some claimed anomalies. The points to emphasize are as follows:

1: The loss field propagates at right angles to the axis of the cable; *ie*, radially into the conductors.

2: The loss field gives rise to the corresponding internal current distribution along the axis of the conductor ($J_\sigma = \sigma E_\sigma$). Note for the loss component that, although the direction of propagation is radial, the $\overline{E}_\sigma$ field is at right angles to the direction of propagation of the radial loss wave, and is along the conductor axis. This induces an axial conduction current, and is the component of current normally experienced.

3: The velocity of propagation within the conductor (copper) is both very slow and frequency-dependent. As a consequence, different frequencies propagate at different velocities; *ie*, the material is highly dispersive.

4: The velocity of the loss field is directly dependent upon σ and μ, which should be noted for magnetic materials. Usual analysis assumes σ to be a smooth and continuous function. However, crystal boundaries suggest discontinuities in σ, such that the conductors appear more like stranded, though disjointed, wire, where such discontinuity represents a point of at least partial reflection and field redistribution. It is anticipated that if the microcurrents within the conductor could be observed, they might well show aspects of chaotic activity.

5: There is a problem even if σ is a linear but discontinuous function. However, nonlinearity due to partial semiconductor diode boundaries could lead to a complicated, frequency-interleaved intermodulation.

6: Stranded conductors without individual strand insulation appear to be a poor construction when viewed by this model, as the loss field propagates against the strands and experiences discontinuities in air/copper boundaries that are inevitably random. This is comparable to a large-scale gran-

4 Hawksford, ibid.

ularity where crystal boundaries possibly represent a similar structure at the microlevel, but within the copper. A single strand of large-crystal copper or multiple strands of insulated wire—the quality of this dielectric will be important—will behave more as a simple impedance.

Conventional theory and actual conductor performance merge: At a diameter of around 0.8mm, the conductor becomes closer to a low-valued ideal resistor at audio frequencies.

7: Irregularities in cable construction and directional wave properties in the dielectric could lead to differences in the $\overline{E}_\sigma$ field patterns, hence current distribution within the conductors depending upon which end is the source. (Perhaps current vortices form, like whirlpools in a stream of water? The Chaos model?)

The exact nature of the loss field would in principle exhibit differences, thus allowing the cable to have a directional characteristic in that the error is not mirror-symmetric. Slight variations in diameter, for example—or, indeed, internal crystal structure—might well occur in manufacture due to a mechanical stress field. Such effects, however, would appear to lie in the domain of errors of errors. They would necessarily be of an extremely subtle nature.

Since boundary conditions, hence the loss field, are indirectly affected by all materials within the cable construction including surface oxidation of the conductor, we would expect each element to contribute to performance.

9: The time taken for the field to propagate to the skin depth δ is longer at low frequencies. Thus, thick conductors would appear more problematic at low frequencies, showing a greater tendency to time dispersion; although, to counter this, the overall error is lower.

10: $\delta = [2/(\omega\mu\sigma)]^{0.5}$. Magnetic conductors therefore have μ-dependent skin depths, and μ will exhibit nonlinear hysteresis. This needs investigation, as it suggests magnetic conductors should be avoided.

11: It appears cable defects have their greatest effects under transient excitation rather than within the pseudo steady-state of sustained tones. Transient edges are effectively time-smeared or broadened (albeit by a small amount), where this dispersion is a function of both the signal and the properties and dimensions of the conductors.

Amplitude frequency-response errors in the steady-state are at a level that is insignificant when listening to steady-state tones. However, their significance when mapped via the error function onto transient signals may well be of greater concern, with possibly a greater significance in stereo listening. In this sense, we support comments made by John Atkinson[5] on the importance of maintaining transient integrity at the beginning and end of sequences of sound, rather than worrying about slight relative level errors in the pseudo–steady-state of a sustained tone, or a slight change in harmonic balance.

It's the old story of measuring a system's frequency and phase responses with insufficient accuracy to extract the true system error, and then misinterpreting the significance of that error: check out the error function.[6]

12: Axial propagation within the dielectric is usually not considered important at audio frequencies, as interconnects are generally much shorter than a wavelength, even at 20kHz. However, we have directed our attention to the loss field *within* the conductors, where, due to the slow velocity, cable dimensions comparable to the wavelength are significant. It is suggested that this viewpoint is usually not considered, where "skin depth" is rarely presented as a propagation or diffusion phenomenon.

Measurements of some loudspeaker cables have confirmed that skin depth must be considered if an accurate estimate of the series impedance is to be made. This is more noticeable for high-capacitance, low- (external) inductance cables where, proportionally, the impedance-related effects of skin depth are a greater fraction of the total series impedance.

5 John Atkinson, "Comment," *HFN/RR*, February & March 1985, Vol.30 Nos.2 & 3.

6 Hawksford, ibid.

Summary

We conclude from these observations that conductors should be sufficiently thin that only a fraction of a wavelength at the highest audio frequency is trapped within the conductors, although multiple, separately insulated (Litz) strands can usefully lower the series impedance due to the internal field in the conductors. The use of either copper/silver tubes or rectangular ribbons represent alternatives to the solid cylindrical conductor, especially if the wall thickness is kept below the critical upper bound (0.4mm to 0.8mm). In these constructions, we can obtain a much higher volume of copper to lower the DC resistance/meter and simultaneously control skin depth to prevent a significant impedance rise over the audio band due to the internal conductor fields. In all cable constructions, however, a narrow dielectric spacing between conductors will reduce the inductance due to the external magnetic field and thus reduce further series impedance, albeit with an increase in shunt capacitance.

Interleaving multiple conductors, both in Litz fashion and for send-and-return conductors, can lower susceptibility to external fields (*eg*, hum pickup) and lower the external series inductive component. This is an interesting balancing trick, especially when combined with EMC requirements. The external propagating fields should be distributed as uniformly as possible over the whole surface of the conductor, and the composite cable should be tightly wrapped to prevent external mechanical vibration from modulating the characteristic impedance (shaking wires, coils, and interconnects in loudspeaker systems, for example).

This article has tried to describe a more rigorous model (finely etched with a little speculation) for cable systems by reviewing some fundamental electromagnetic principles. It is important not to make engineering simplifications too prematurely when evolving a model. We have made some approximations, however, as field patterns can be highly complicated and depend upon cable geometry and internal material behavior at a molecular level. (I keep thinking of current vortices.) Nevertheless, there is sufficient evidence to suggest that a cable's performance is not as simple as it first appears, often because the operation is viewed too simplistically, and because our notions of lumped circuit elements (discrete R, C, and L) warp our thinking, especially with respect to skin depth.

For me, the most striking observation is the slow, frequency-dependent velocity of a wave traveling in a conductor. Also, high conductivity and permeability make the conductor appear much larger on the inside, and crystal boundaries act as partitions within that space. TARDIS, Transient And Resistance DIStortion—watch out for the Steven Spielberg version!

References & further reading

[1] Magid, L.M., *Electromagnetic Fields, Energy and Waves*, John Wiley and Sons, Inc. ISBN 0-471-56334-X, 1972.

[2] Skilling, H.H., *Fundamentals of Electric Waves*, John Wiley and Sons, Inc. 1948.

[3] Larrain, P. and Corson, D., *Electromagnetic Fields and Waves*, W.H. Freeman and Company, ISBN 0-7167-0331-9, 1962.